Applied Mechanics Workbook

A Concise Introduction to Computational Tools

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Preface

A Concise Introduction to Computational Tools provides a series of tutorials derived from lecture notes, offering clear and focused guidance on essential topics. No prior programming experience is required.

Students are expected to be comfortable with college-level mathematics and science and are strongly encouraged to consult reliable reference texts. For example, foundational and practical texts in mathematics, applied mechanics and computational tools, including Bird (2021), Hannah & Hillier (1995), Russell et al. (2021), Bolton (2021), Shaw (2017), Sweigart (2021), and Sweigart (2020).

The tutorials also assume working familiarity with either macOS or Microsoft Windows. For best results, these materials should be used on a laptop or desktop computer rather than a tablet or smartphone.

Chapter 1

Units, Vectors, and Scalar Quantities

1.1 SI Units

The book adopts the **Système International (SI)** of units. The base units most relevant to mechanics are:

Quantity	Unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Force	newton	N
Energy / Work	joule	J
Power	watt	W
Pressure / Stress	pascal	Pa

Derived units follow from these. For example:

$$1\ \mathrm{N} = 1\ \mathrm{kg} \cdot \mathrm{m/s}^2 \qquad 1\ \mathrm{Pa} = 1\ \mathrm{N/m}^2 \qquad 1\ \mathrm{J} = 1\ \mathrm{N} \cdot \mathrm{m}$$

1.2 Scalar and Vector Quantities

Scalar quantities have magnitude only and obey ordinary arithmetic:

- Examples: mass, speed, energy, temperature, time

Vector quantities have both magnitude and direction and must be combined using vector algebra:

- Examples: force, velocity, acceleration, displacement, momentum

A vector is represented graphically as an arrow — its length denotes magnitude, its orientation denotes direction.

1.3 Vector Addition

1.3.1 Triangle Rule

Two vectors **A** and **B** are added tip-to-tail; the resultant **R** closes the triangle:

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

1.3.2 Parallelogram Rule

Both vectors are drawn from the same point; the resultant is the diagonal of the parallelogram they form.

1.3.3 Polygon Rule

For more than two vectors, they are placed tip-to-tail in sequence; the resultant closes the polygon from the tail of the first to the tip of the last.

1.4 Vector Subtraction

Subtracting **B** from **A** is equivalent to adding the negative of **B**:

$$\mathbf{R} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

1.5 Resolution of Vectors

Any vector **F** can be resolved into two perpendicular components. For a vector at angle θ to the horizontal:

$$F_x = F \cos\theta$$

$$F_y = F \sin\theta$$

This is the most widely used technique in the book — resolving all forces into horizontal and vertical components before applying equilibrium conditions.

1.6 Resultant of Several Vectors

To find the resultant of a system of vectors analytically:

1. Resolve each vector into x and y components
2. Sum all components in each direction:

$$\Sigma F_x = F_{1x} + F_{2x} + \cdots \qquad \Sigma F_y = F_{1y} + F_{2y} + \cdots$$

3. Find the magnitude of the resultant:

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

4. Find the direction:

$$\theta = \arctan\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

1.7 The Laws of Sines and Cosines

For cases involving triangles of vectors, these laws provide an analytical alternative to graphical methods.

Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

1.8 Summary of Equations

Concept	Equation
Horizontal component	$F_x = F\cos\theta$
Vertical component	$F_y = F\sin\theta$
Resultant magnitude	$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$
Resultant direction	$\theta = \arctan(\Sigma F_y/\Sigma F_x)$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$

1.9 Summary

The resolution-and-recombination technique — breaking every vector into perpendicular components, summing each direction separately, then reconstructing the resultant — is the single most important skill introduced in this chapter. It underpins equilibrium analysis, dynamics, stress transformation, and virtually every other topic in the book.

Chapter 2

Balancing

2.1 Static Balance

A system is statically balanced when the resultant of all centrifugal forces due to rotating masses is zero — i.e., the centre of mass lies on the axis of rotation.

This is checked by ensuring the vector sum of all mr terms equals zero:

$$\sum mr = 0$$

A single correction mass placed in the same plane can achieve static balance.

2.2 Dynamic Balance

Static balance alone is insufficient for systems with masses distributed across different axial planes. Dynamic balance requires two conditions to be satisfied simultaneously:

1. The vector sum of all centrifugal forces equals zero (static condition)
2. The vector sum of all **moments** of these forces about any reference plane equals zero

$$\sum mr = 0 \qquad \text{and} \qquad \sum mrl = 0$$

This generally requires **two correction masses** placed in two chosen planes.

2.3 Balancing in a Single Plane

For coplanar rotating masses, a graphical or analytical approach uses the **force polygon**. Closing the mr vector polygon determines the required balancing mass and its angular position.

2.4 Balancing in Several Planes

For masses distributed along a shaft, the procedure is:

1. Choose a reference plane and construct a **couple polygon** using mrl terms, where l is the axial distance from the reference plane
2. Closing the couple polygon gives the first correction mass (magnitude and angle)
3. Construct the **force polygon** using mr terms to find the second correction mass

2.5 Balancing of Reciprocating Masses

2.5.1 Primary and Secondary Forces

The piston acceleration in a crank-slider mechanism gives rise to two force components:

Primary force (first-order):

$$F_{\text{primary}} = mr\omega^2 \cos\theta$$

Secondary force (second-order):

$$F_{\text{secondary}} = \frac{mr\omega^2 \cos 2\theta}{n}$$

where $n = \ell/r$ is the ratio of connecting rod length ℓ to crank radius r.

2.5.2 Multi-Cylinder Engines

In multi-cylinder in-line engines, the angular phasing of the cranks is arranged to cancel primary and secondary forces and couples. The chapter examines which engine configurations achieve complete or only partial balance.

2.6 Partial Balance

Complete balance is not always possible or practical — particularly for single-cylinder engines. In such cases:

- A balance mass is added to **reduce** the dominant unbalanced force
- A residual force in one direction is accepted to eliminate the more harmful force in the perpendicular direction

2.7 Summary of Equations

Concept	Equation
Centrifugal force	$F = mr\omega^2$
Static balance condition	$\sum mr = 0$
Dynamic balance condition	$\sum mr = 0$ and $\sum mrl = 0$
Primary reciprocating force	$F = mr\omega^2 \cos\theta$
Secondary reciprocating force	$F = mr\omega^2 \cos 2\theta \,/\, n$

2.8 Summary

Balancing is fundamentally a **vector problem**. Both graphical (polygon) and analytical methods are used throughout. Dynamic balancing always requires satisfying both force *and* moment equilibrium, and the chapter builds progressively from simple single-plane cases up to the complexity of multi-cylinder engine analysis.

Chapter 3

Simple Harmonic Motion

Simple Harmonic Motion: The foundation of vibration analysis. A particle undergoes SHM when its acceleration is proportional to displacement and directed toward the equilibrium position:

$$a = -\omega^2 x$$

Key relationships — displacement, velocity, acceleration, amplitude, period, and frequency — are all derived from this condition.

3.1 Free Undamped Vibration

3.1.1 Mass-Spring System

The simplest model: a mass m on a spring of stiffness k:

$$\omega_n = \sqrt{\frac{k}{m}} \qquad f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

3.1.2 Simple Pendulum

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

3.1.3 Equivalent Stiffness

Springs in series and parallel are reduced to a single equivalent stiffness before applying the standard formulae:

$$k_{\text{series}} = \frac{k_1 k_2}{k_1 + k_2} \qquad k_{\text{parallel}} = k_1 + k_2$$

3.2 Energy Method (Rayleigh's Method)

An alternative to force methods — equating maximum kinetic energy to maximum potential energy to find the natural frequency. Particularly useful where writing equations of motion is complex:

$$\frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} k x_{\text{max}}^2$$

3.3 Damped Free Vibration

In practice, energy is dissipated. A viscous damping force proportional to velocity is assumed:

$$F_d = -c\dot{x}$$

The **damping ratio** ζ determines the character of the motion:

$$\zeta = \frac{c}{2\sqrt{mk}}$$

Condition	Behaviour
$\zeta < 1$	Underdamped — oscillates with decaying amplitude
$\zeta = 1$	Critically damped — returns to rest without oscillation
$\zeta > 1$	Overdamped — sluggish return to rest

The **damped natural frequency** is reduced from the undamped value:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

3.4 Forced Vibration (Undamped)

When a harmonic exciting force $F = F_0 \sin \omega t$ is applied, the steady-state amplitude is:

$$X = \frac{F_0/k}{1 - (\omega/\omega_n)^2}$$

Resonance occurs when $\omega \to \omega_n$, causing theoretically infinite amplitude.

3.5 Forced Vibration (Damped)

Damping limits the amplitude at resonance. The **dynamic magnification factor (DMF)** gives the ratio of dynamic to static deflection:

$$\text{DMF} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

At resonance ($\omega = \omega_n$), the DMF reduces to:

$$\text{DMF}_{\text{resonance}} = \frac{1}{2\zeta}$$

3.6 Whirling of Shafts

A rotating shaft has a **critical speed** at which it deflects violently — directly analogous to resonance. The critical (whirling) speed corresponds to the natural frequency of transverse vibration:

$$N_c = \frac{30}{\pi}\sqrt{\frac{g}{\delta_{st}}}$$

where δ_{st} is the static deflection of the shaft under its own (or the rotor's) weight. Operating speeds should be well clear of N_c.

3.7 Summary of Equations

Concept	Equation
SHM condition	$a = -\omega^2 x$
Natural frequency (spring-mass)	$\omega_n = \sqrt{k/m}$
Natural frequency (pendulum)	$f_n = \frac{1}{2\pi}\sqrt{g/L}$
Damping ratio	$\zeta = c\,/\,2\sqrt{mk}$
Damped natural frequency	$\omega_d = \omega_n\sqrt{1-\zeta^2}$
Resonance condition	$\omega = \omega_n$
DMF (damped forced)	$1/\sqrt{[1-r^2]^2 + [2\zeta r]^2}$, where $r = \omega/\omega_n$
Critical shaft speed	$N_c = (30/\pi)\sqrt{g/\delta_{st}}$

3.8 Summary

All vibration problems reduce to understanding the interplay between **stiffness** (restoring force), **mass** (inertia), and **damping** (energy dissipation). Resonance avoidance and the role of damping in controlling amplitudes are the central practical concerns for mechanical design.

Chapter 4

Stress & Strain

4.1 Direct Stress and Strain

When a bar is subjected to an axial force F over cross-sectional area A, the direct stress is:

$$\sigma = \frac{F}{A}$$

The resulting axial strain is the ratio of extension to original length:

$$\varepsilon = \frac{\delta L}{L}$$

4.2 Young's Modulus (Modulus of Elasticity)

Within the elastic limit, stress and strain are proportional — **Hooke's Law**:

$$E = \frac{\sigma}{\varepsilon}$$

This gives the deformation of a bar under axial load:

$$\delta L = \frac{FL}{AE}$$

4.3 Shear Stress and Shear Strain

Shear stress acts tangentially across a plane:

$$\tau = \frac{F}{A}$$

Shear strain γ is the angular deformation (in radians). The **Modulus of Rigidity** (shear modulus) relates them:

$$G = \frac{\tau}{\gamma}$$

4.4 Poisson's Ratio

Axial loading causes lateral contraction (or expansion) as well as axial strain. Poisson's ratio ν relates the two:

$$\nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{axial}}}$$

Typical values for metals: $\nu \approx 0.25$–0.33.

4.5 Volumetric Strain and Bulk Modulus

Under hydrostatic (equal all-round) stress, the volumetric strain is:

$$\varepsilon_v = \frac{\delta V}{V}$$

The **Bulk Modulus** K relates hydrostatic stress to volumetric strain:

$$K = \frac{\sigma}{\varepsilon_v}$$

4.6 Relationship Between Elastic Constants

The three elastic constants E, G, and K are not independent — they are linked through Poisson's ratio:

$$E = 2G(1 + \nu)$$

$$E = 3K(1 - 2\nu)$$

4.7 Composite Bars

When two materials are bonded together (e.g. a steel rod inside a brass tube), two equations are needed:

Compatibility — both materials deform by the same amount:

$$\delta L_1 = \delta L_2 \quad \Rightarrow \quad \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

Equilibrium — internal forces sum to the applied load:

$$\sigma_1 A_1 + \sigma_2 A_2 = F$$

Solving these simultaneously gives the stress in each material.

4.8 Thermal Stress

If a bar is **prevented from expanding freely** when heated, a compressive stress is induced:

$$\sigma = E\alpha\Delta T$$

where:

- α = coefficient of linear thermal expansion
- ΔT = temperature change

For a composite bar with mismatched expansion, the same compatibility and equilibrium approach applies.

4.9 Factor of Safety

The factor of safety relates the failure stress to the permissible working stress:

$$\text{FoS} = \frac{\text{failure stress}}{\text{allowable (working) stress}}$$

A higher FoS reflects greater uncertainty in loading or material properties.

4.10 Summary of Equations

Concept	Equation
Direct stress	$\sigma = F/A$
Direct strain	$\varepsilon = \delta L/L$
Young's modulus	$E = \sigma/\varepsilon$
Axial deformation	$\delta L = FL/AE$
Shear modulus	$G = \tau/\gamma$
Poisson's ratio	$\nu = -\varepsilon_{\text{lat}}/\varepsilon_{\text{axial}}$
Bulk modulus	$K = \sigma/\varepsilon_v$
Elastic constants	$E = 2G(1+\nu) = 3K(1-2\nu)$
Thermal stress	$\sigma = E\alpha\Delta T$

4.11 Summary

This chapter establishes the language of solid mechanics — stress, strain, and the elastic constants — that underpins all subsequent structural chapters. The key skill is setting up **compatibility** and **equilibrium** equations correctly for statically indeterminate problems such as composite bars and thermally loaded members.

Chapter 5

Torsion

5.1 Assumptions of Simple Torsion Theory

The theory applies to circular cross-sections (solid or hollow) and assumes:

- The material is homogeneous and isotropic
- Plane cross-sections remain plane after twisting
- Shear stress is proportional to shear strain (elastic behaviour)
- The shaft is straight and of uniform cross-section

5.2 The Torsion Formula

The fundamental relationship linking shear stress, torque, geometry, and angle of twist:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

where:

- T = applied torque (N · m)
- J = polar second moment of area (m)
- τ = shear stress at radius r (Pa)
- G = modulus of rigidity (Pa)
- θ = angle of twist (radians)
- L = length of shaft (m)

5.3 Polar Second Moment of Area

For a **solid circular shaft** of diameter d:

$$J = \frac{\pi d^4}{32}$$

For a **hollow circular shaft** with outer diameter d_o and inner diameter d_i:

$$J = \frac{\pi(d_o^4 - d_i^4)}{32}$$

5.4 Shear Stress Distribution

Shear stress varies **linearly** from zero at the centre to a maximum at the outer surface:

$$\tau_{\text{max}} = \frac{T \cdot r}{J} = \frac{16T}{\pi d^3} \quad \text{(solid shaft)}$$

A hollow shaft is more efficient — it carries almost the same torque for less material mass.

5.5 Angle of Twist

The total angle of twist along a shaft of length L:

$$\theta = \frac{TL}{GJ}$$

For a stepped shaft (different diameters along its length), the total twist is the sum of twists in each segment:

$$\theta_{\text{total}} = \sum \frac{T_i L_i}{GJ_i}$$

5.6 Power Transmission

The relationship between transmitted power, torque, and rotational speed:

$$P = T\omega = \frac{2\pi NT}{60}$$

where N is rotational speed in rpm and ω is in rad/s. This is used to find the required shaft diameter for a given power and speed.

5.7 Composite (Compound) Shafts

5.7.1 Shafts in Series

The same torque passes through each section; total twist is the sum of individual twists:

$$\theta_{\text{total}} = \frac{TL_1}{GJ_1} + \frac{TL_2}{GJ_2}$$

5.7.2 Shafts in Parallel

Both shafts share the applied torque and twist by the same angle:

$$T = T_1 + T_2 \qquad \theta_1 = \theta_2$$

These two conditions give the equations needed to solve for the torque in each shaft.

5.8 Close-Coiled Helical Springs

A close-coiled helical spring under axial load W is primarily a torsion problem. The wire is subjected to torque $T = WR$, where R is the mean coil radius.

Shear stress in the wire:

$$\tau = \frac{16WR}{\pi d^3}$$

Axial deflection:

$$\delta = \frac{64WR^3n}{Gd^4}$$

Stiffness of the spring:

$$k = \frac{W}{\delta} = \frac{Gd^4}{64R^3n}$$

where d is the wire diameter and n is the number of active coils.

5.9 Summary of Equations

Concept	Equation
Torsion formula	$T/J = \tau/r = G\theta/L$
Polar moment — solid shaft	$J = \pi d^4/32$
Polar moment — hollow shaft	$J = \pi(d_o^4 - d_i^4)/32$
Max shear stress (solid)	$\tau_{\max} = 16T/\pi d^3$
Angle of twist	$\theta = TL/GJ$
Power transmitted	$P = 2\pi NT/60$
Spring deflection	$\delta = 64WR^3n/Gd^4$
Spring stiffness	$k = Gd^4/64R^3n$

5.10 Summary

Torsion analysis always begins with the torsion formula $T/J = \tau/r = G\theta/L$. For composite shaft problems, correctly identifying whether shafts are in **series** (same torque) or **parallel** (same twist) is the critical first step. The close-coiled helical spring is a direct application of torsion theory to a practical engineering component.

Chapter 6

Combined Stress

Chapter 7

Fluid Mechanics

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Attributions

This book incorporates figures, data, and other resources from the following sources:

- Figures in the formulae, the centroids and second moments of area, are adapted from the Wikipedia pages on Centroids and Second moments of area.

Colophon

This book was typeset with Quarto 1.9.27 using XeLaTeX and Latin Modern fonts, with Menlo for code blocks.

Technical Notes

The following tools and technologies were used in the preparation of this book:

- Python: Primary language for computations
- uv: Modern Python package manager and resolver
- Ruff: Fast Python linter and code formatter
- Black: Python code formatter
- Bash: Unix shell for task automation
- macOS: Main development operating system

Appendix A

SI System Common Mistakes

Using the SI system correctly is crucial for clear communication in science and engineering. Below are common mistakes in using the SI system, examples of incorrect usage, and how to correct them.

Table A.1: SI system rules and common mistakes

Concept	Mistake	Correct Usage	Notes
Use of SI Unit Symbols	`m./s`	`m/s`	Use the correct format without additional punctuation.
Spacing Between Value & Unit	`10kg`	`10 kg`	Always leave a space between the number and the unit symbol.
Incorrect Unit Symbols	`sec`, `hrs`, `°K`	`s`, `h`, `K`	Use the proper SI symbols; symbols are case-sensitive.
Abbreviations for Units	`5 kilograms (kgs)`	`5 kilograms (kg)`	Avoid informal abbreviations like "kgs"; adhere to standard symbols.

Concept	**Mistake**	**Correct Usage**	**Notes**
Multiple Units in Expressions	5 m/s/s, 5 kg/meter^2	5 m/s^2, 5 kg/m^2	Use compact, standardized formats for derived units.
Incorrect Use of Prefixes	0.0001 km	100 mm	Choose prefixes to keep numbers in the range (0.1 x < 1000).
Misplaced Unit Symbols	5/s, kg10	5 s^{-1}, 10 kg	Symbols must follow numerical values, not precede them.
Degrees Celsius vs. Kelvin	300°K	300 K	Kelvin is written without "degree"
Singular vs. Plural Units	5 kgs, 1 meters	5 kg, 1 meter	Symbols do not pluralize; full unit names follow grammar rules.
Capitalization of Symbols	Kg, S, Km, MA	kg, s, km, mA	Symbols are case-sensitive; use uppercase only where specified (e.g., N, Pa).
Capitalization of Unit Names	Newton, Pascal, Watt	newton, pascal, watt	Unit names are lowercase, even if derived from a person's name, unless starting a sentence.
Prefix Capitalization	MilliMeter, MegaWatt	millimeter, megawatt	Prefixes are lowercase for (10^{-1}) to (10^{-9}), uppercase for (10^6) and larger (except k for kilo).
Formatting in Reports	5, Temperature: 300	5 kg, Temperature: 300 K	Always specify units explicitly.

Appendix B

Greek Letters

The following tables present the names of Greek letters and selected symbols commonly used in engineering courses, ensuring precise reference and avoiding reliance on informal descriptors such as “squiggle.”

Table B.1: Greek letters.

Lower Case	Upper Case	Name
α	A	alpha
β	B	beta
γ	Γ	gamma
δ	Δ	delta
ϵ	E	epsilon
ζ	Z	zeta
η	E	eta
θ	Θ	theta
ι	I	iota
κ	K	kappa
λ	Λ	lambda
μ	M	mu
ν	N	nu
ξ	Ξ	xi
o	O	omicron
π	Π	pi
ρ	P	rho
σ	Σ	sigma
τ	T	tau
υ	Υ	upsilon
ϕ	Φ	phi
χ	X	chi

Lower Case	Upper Case	Name
ψ	Ψ	psi
ω	Ω	omega

Table B.2: Commonly used symbols in engineering courses.

Symbol	Name	Use	Course
Δ	Delta	Change	Thermodynamics
Δ	Delta	Displacement	Naval Architecture
∇	Nabla	Volume	Naval Architecture
Σ	Sigma	Sum	Thermodynamics, Naval Architecture, Applied Mechanics
σ	Sigma	Stress	Thermodynamics, Applied Mechanics
ϵ	Epsilon	Modulus of elasticity	Thermodynamics, Applied Mechanics
η	Eta	Efficiency	Thermodynamics
μ	Mu	Friction	Thermodynamics, Applied Mechanics
ω	Omega	Angular velocity	Thermodynamics, Applied Mechanics
ρ	Rho	Density	Thermodynamics, Naval Architecture
τ	Tau	Torque	Thermodynamics, Applied Mechanics